

## A STUDY ON NANO TOPOLOGY

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### Keywords:

### Abstract

Nano topology,  
Nano open set,  
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graph.

The aim of this paper is to study the basic concept of Nano topology. And its application in real life situation is also discussed through an example.

## I. INTRODUCTION

The theory of Nano topology [3] proposed by Lellis Thivagar and Richard is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The elements of a Nano topological space are called the Nano open set. It originates from the Greek word 'Nanos' which means 'dwarf' in its modern scientific sense, an order of magnitude-one billionth. The Topology is named as Nano topology so because of its size, since it has at most five elements. The author has defined Nano topological space in terms of Lower and upper approximations. He also introduced certain weak form of Nano open set[3] such as Nano  $\alpha$  open set, Nano semi-open sets and nano pre open sets. Further he introduced continuity [4] which is the core concept of topology in Nano topological space.

## 2. PRELIMINARIES

**Definition:3.1** [2] Let  $U$  be a non empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the

indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ ,

1. The Lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for

2. certain classified as  $X$  with respect to  $R$  and is defined by  $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ . Where  $R(x)$  denotes the equivalence class determined by  $x$ .

3. The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and is defined by  $u_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$ .

4. The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither as  $X$  nor as not-  $X$  with respect to  $R$  and is defined by  $B_R(X) = u_R(X) - L_R(X)$ .

**Definition:3.2**[2] Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \phi, L_R(X), u_R(X), B_R(X)\}$

where  $X \subseteq U$ . Then  $\tau_R(X)$  satisfies the following axioms.

1.  $U$  and  $\phi \in \tau_R(X)$ .
2. The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .
3. The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

That is  $\tau_R(X)$  forms a topology on  $U$  called as the Nano topology on  $U$  with respect to  $X$ . We call  $(U, \tau_R(X))$  as the Nano topological space. The elements of  $\tau_R(X)$  are called as Nano open sets.

**Example : 3.3** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{b\}, \{c, d\}\}$  and  $X = \{a, c\} \subset U$ . Then the Nano topology is  $\tau_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ .

### 3. APPLICATION OF NANO TOPOLOGY

#### Definition:3.1 [5]

A graph  $G$  is an ordered pair of disjoint sets  $(V, E)$ , Where  $V$  is nonempty set of vertices and  $E$  is set of edges.

#### Definition 3.2[5]

Let  $G(V, E)$  be a graph,  $v \in V(G)$ . Then we define the neighbourhood of 'v' as follows,

$$N(v) = \{v\} \cup \{u \in V(G) : uv \in E(G)\}.$$

#### Definition 3.3[5]

Let  $G(V, E)$  be a graph,  $H$  is a subgraph of  $G$  and  $N(v)$  be neighbourhood of  $v$  in  $V$ . Then we define

(i) The lower approximation operation as follows:  $L: P[V(G)] \rightarrow P[V(G)]$  such that,

$$L_N[V(H)] = \bigcup_{v \in V(G)} \{v \mid N(v) \subseteq V(H)\}.$$

(ii) The upper approximation operation as follows:  $U: P[V(G)] \rightarrow P[V(G)]$  such that,

$$U_N[V(H)] = \{N(v) : v \in V(H)\}.$$

(iii) The boundary region is defined as  $B_M[V(H)] = U_N[V(H)] - L_N[V(H)]$ .

#### Definition:3.4 [5]

Let  $G$  be a graph,  $N(v)$  be neighbourhood of  $v$  in  $V$  and  $H$  be a subgraph of  $G$ ,  $\tau_N[V(H)] = \{V(G), \phi, L_N[V(H)], U_N[V(H)], B_N[V(H)]\}$  forms a topology on  $V(G)$  called the nanotopology on  $V(G)$  with respect to  $V(H)$ . We call  $[V(G), \tau_N[V(H)]]$  as the nanotopological space induced by a graph.

#### Theorem 3.6 [5]

Let  $G = [V, E]$  and  $G' = [V', E']$  be any two isomorphic graphs then there exists a homeomorphism  $\phi: [V(G), \tau_N[V(H)]] \rightarrow [V(G'), \tau_N[V(H)]]$  for every subgroup  $H$  of  $G$ .

#### Proof:

Since  $G$  and  $G'$  are isomorphic by defn. there is an isomorphism  $f: V[G] \rightarrow V[G']$  between their Underlying graphs that preserves the direction of each edge and also  $N(x) = N(f(x)) \forall x \in V(G)$ . Suppose  $[V(G), \tau_N[V(H)]]$  and  $[V(G'), \tau_N f([V(H)])]$  be the nano topological space generated by  $V(H)$  and  $f(V(H))$ . Since  $f$  is a bijection clearly, it follows that  $\phi$  is 1-1 and onto.

(i) To prove that  $\phi$  is an open map. Let  $A$  be any nano-open set in  $\tau_N[V(H)]$ , then  $\phi(A)$  is nano-open since  $N(x) = N(f(x)) \forall x \in A$ .

(ii) To prove that  $\phi$  is continuous. Let  $B$  be any nano-open set in  $\tau_N f([V(H)])$ , then  $\phi^{-1}(B)$  is nanoopen in  $\tau_N[V(H)]$  Thus  $\phi$  is a homeomorphism.

#### APPLICATION

Most real-life situations need some sort of approximation to fit mathematical models. The

beauty of using nanotopology in approximation is achieved via approximation for qualitative sub graphs without coding or using assumption. We believe that nanotopological graph structure will be an important base for modification of knowledge extraction of processing.

Graphical isomorphism is a related task for deciding when two graphs with different specifications are structurally equivalent, that is whether they have the same pattern of connections. Nano homeomorphism between two Nano topological spaces are said to be topologically equivalent.

Based on the structural equivalent of graphs and the corresponding nanotopology induced by them, we can check whether the chip produced by a company have striking operational similarity produced by the another company.

**Step 1:** Given the electrical circuit of the chips manufactured by two companies, An electrical network is an interconnection of electrical network elements such as resistances, capacitances, inductances, voltage and current sources, etc., We also assign reference direction by a directed edge results in the directed graph representing the network.

**Step 2:** Convert the electrical circuits  $c_1$  and  $c_2$  into graphs  $G_1$  and  $G_2$

**Step 3:** Check whether  $G_1$  and  $G_2$  are isomorphic, and their corresponding

nanotopologies induced from their vertices are homeomorphic.

**Step 4:** If  $G_1 \cong G_2$  and  $[V(G), \tau_N[V(H)]] \cong [V(G'), \tau_N[f(V(H))]]$  then the corresponding circuits have striking operational similarities.

**Step 5:** Otherwise, we can conclude that both the chip produced are entirely different.

## REFERENCES

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