Nehru E- Journal
A Journal of Nehru Arts and Science College (Autonomous)
Journal Home Page: http://nehrucolleges.net/jo.html
ISSN: 2349-9052

Research Article
Vol 5 issue 1(2017)
(Jan - July 2017)

CORDIAL LABELING OF $K_{n,n}$ RELATED GRAPHS

P.Premkumar[1] and S.Mohammed Shanawaz[2], Asst. Prof., Dept. Of Maths,
Nehru Arts and Science College, Coimbatore 641 105, India

Keywords:

Cloud storage,
Auditing,
Data integrity,
third party
auditor

Abstract

In this article we prove that the star of $K_{n,n}$ is cordial for all $n \geq 2$.

Introduction:

For the basic definitions we follow J.A.Bondy and U.S.R.Murty and Frank Harary[4]. The main definitions are followed by G.V.Ghodasara et al. [7].

Key words: Bipartite graph, Star graph, Star of a graph, Cordial labeling.

Theorem 3.1: Star of complete bipartite graph $K_{n,n}$ is cordial.

Proof: Let $G = (K_{n,n})^*$ be the star of complete bipartite graph $K_{n,n}$.

Let V_1 and V_2 be the partitions of the vertex set of the central copy of complete bipartite graph $K_{n,n}$ in G. Let $v_1, v_2, v_3, \ldots, v_n$ be successive vertices of the set V_1 and $v_{n+1}, v_{n+2}, v_{n+3}, \ldots v_{2n}$ be successive vertices of the set V_2 (in counter clockwise direction).

Let V_{i1} and V_{i2} be the partitions of the vertex set of i^{th} copy of complete bipartite graph $K_{n,n}$ in G (except central one). Let $u_{i1}, u_{i2}, u_{i3}, \ldots, u_{in}$ be successive vertices of the set V_{i1} and let $u_{i(n+1)}, u_{i(n+2)}, u_{i(n+3)}, \ldots, u_{i(2n)}$ be successive vertices of the set V_{i2}.

Let $e_i = u_{i1}v_i$ be the edge joining central copy and i^{th} copy of $K_{n,n}$. Moreover let ${u_{ij}}^{(0)}$ denote the vertices of the copy of $K_{n,n}$ which is joined to vertex with label 0 of the central copy of $K_{n,n}$ by an edge and let ${u_{ij}}^{(1)}$ denote the vertices of the copy of $K_{n,n}$ which is joined to vertex with label 1 of the central copy of $K_{n,n}$, where $i = 1,2,3, \ldots, 2n; j = 1,2,3, \ldots, 2n$.

To define required labeling $f: V(G) \rightarrow \{0,1\}$ we consider the following cases:

Case 1: $n \equiv 1,3 (mod\ 4)$

For $1 \leq i \leq 2n$,

$$f(v_i) = \begin{cases} 0 & ; if\ i \equiv 2,3 (mod\ 4) \\ 1 & ; if\ i \equiv 0,1 (mod\ 4) \end{cases}$$

$$f(u_{ij})^{(0)} = \begin{cases} 0 & ; if\ j \equiv 0,1 (mod\ 4) \\ 1 & ; if\ j \equiv 2,3 (mod\ 4) \end{cases} \quad ; 1 \leq j \leq 2n$$

$$f(u_{ij})^{(1)} = \begin{cases} 0 & ; if\ j \equiv 2,3 (mod\ 4) \\ 1 & ; if\ j \equiv 0,1 (mod\ 4) \end{cases} \quad ; 1 \leq j \leq 2n$$

Case 2: $n \equiv 0,2 (mod\ 4)$

For $1 \leq i \leq 2n$,

$$f(v_i) = \begin{cases} 0 & ; if\ i \equiv 2,3 (mod\ 4) \\ 1 & ; if\ i \equiv 0,1 (mod\ 4) \end{cases}$$

$$f(u_{ij}) = \begin{cases} 0 & ; if\ j \equiv 0,2 (mod\ 4) \\ 1 & ; if\ j \equiv 1,3 (mod\ 4) \end{cases} \quad ; 1 \leq j \leq 2n$$

The graph G under consideration satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table I. Hence the graph G is cordial graph.

Let $n = 4a + b$ where $n \in N$; $a \in W$; $b \in \{0,1,2,3\}$.

TABLE: I

1.1.1. b	*1.1.2. Vertex conditions*	*1.1.3. Edge conditions*
1.1.4. 1,3	*1.1.5.* $v_f(0) = v_f(1)$	*1.1.6.* $1 + e_f(0) = e_f(1)$
1.1.7. 0,2	*1.1.8.* $v_f(0) = v_f(1)$	*1.1.9.* $e_f(0) = e_f(1)$

Illustration 3.1:

The cordial labeling of star to complete bipartite graph $K_{3,3}$ is shown in figure 3.1(b) as an illustration for the proof of Theorem 3.1. It is the case related to $n \equiv 3(mod\ 4)$.

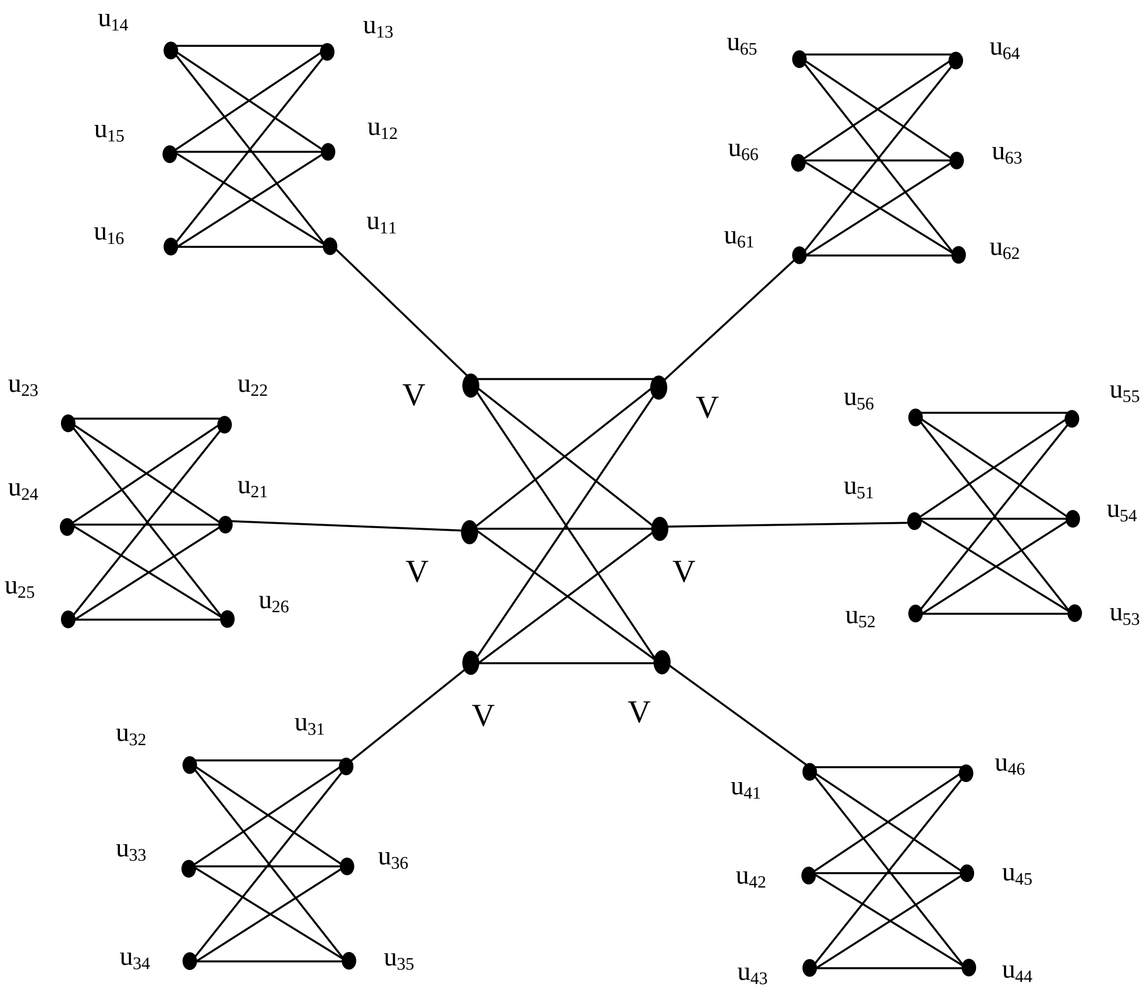

Figure 3.1(a)

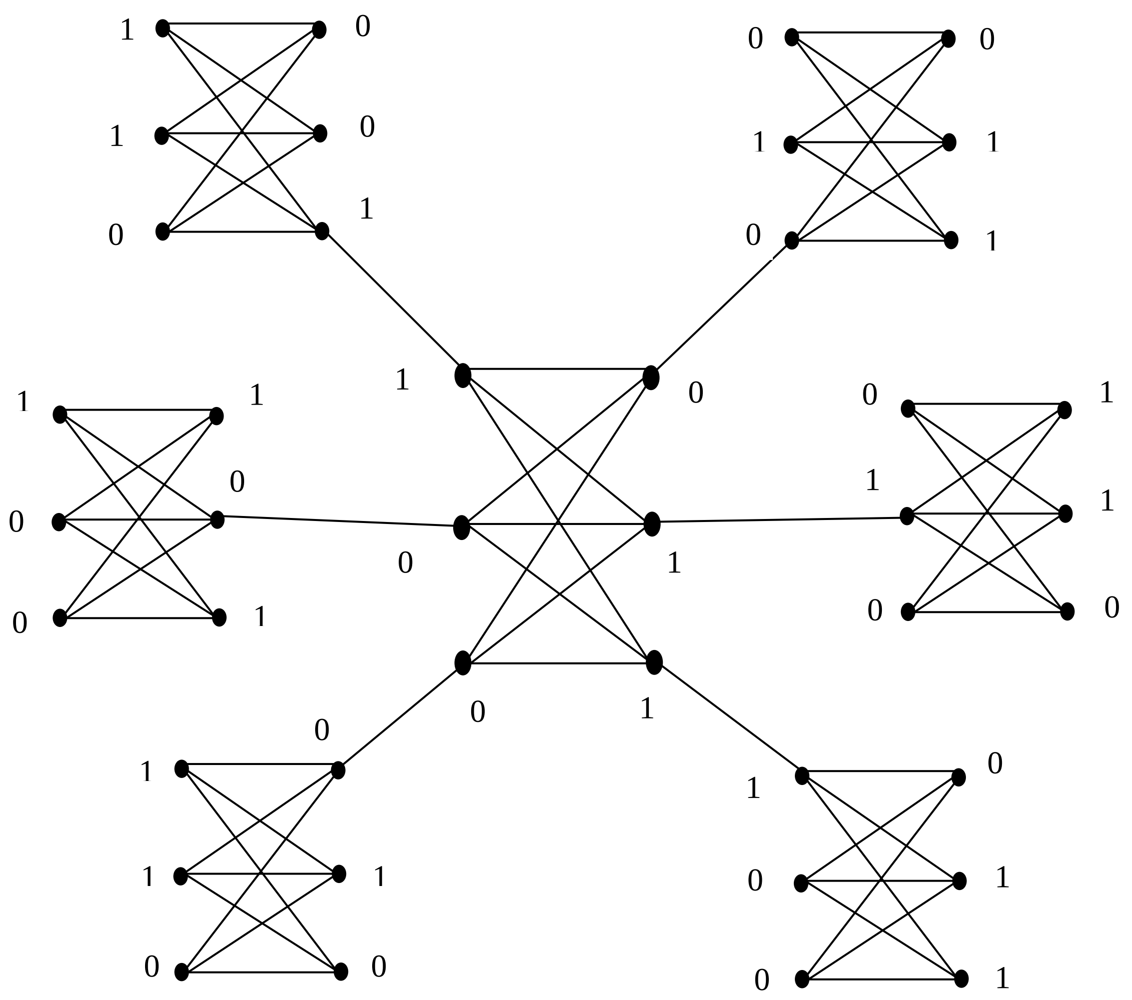

Figure 3.1(b) : Cordial labeling of star of complete bipartite graph $K_{3,3}$

Here $v_f(0) = 21 = v_f(1)$, $e_f(0) = 34$ and $e_f(1) = 35$.

Thus $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence the above graph is cordial.

Illustration 3.2:

The cordial labeling of star to complete bipartite graph $K_{5,5}$ is shown in figure 3.2 as an illustration for the proof of Theorem 1. It is the case related to $n \equiv 1(mod\ 4)$.

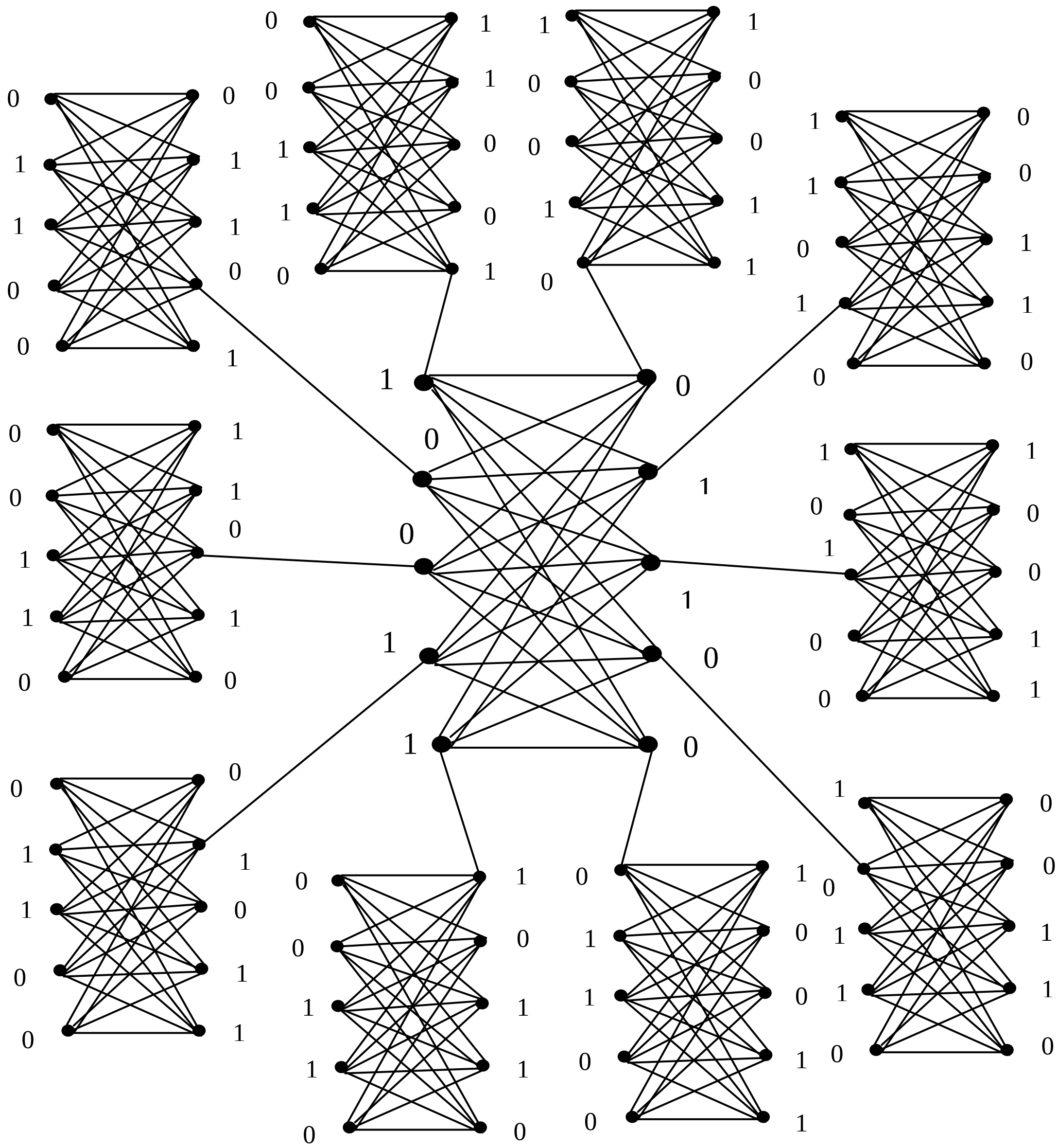

Figure 3.2 : Cordial labeling of star of complete bipartite graph $K_{5,5}$

Here $v_f(0) = 55 = v_f(1)$, $e_f(0) = 142$ and $e_f(1) = 143$.

Thus $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence the above graph is cordial.

Illustration 3.3:

The cordial labeling of star to complete bipartite graph $K_{2,2}$ is shown in figure 3.3 as an illustration for the proof of Theorem 1. It is the case related to $n \equiv 2(mod\ 4)$

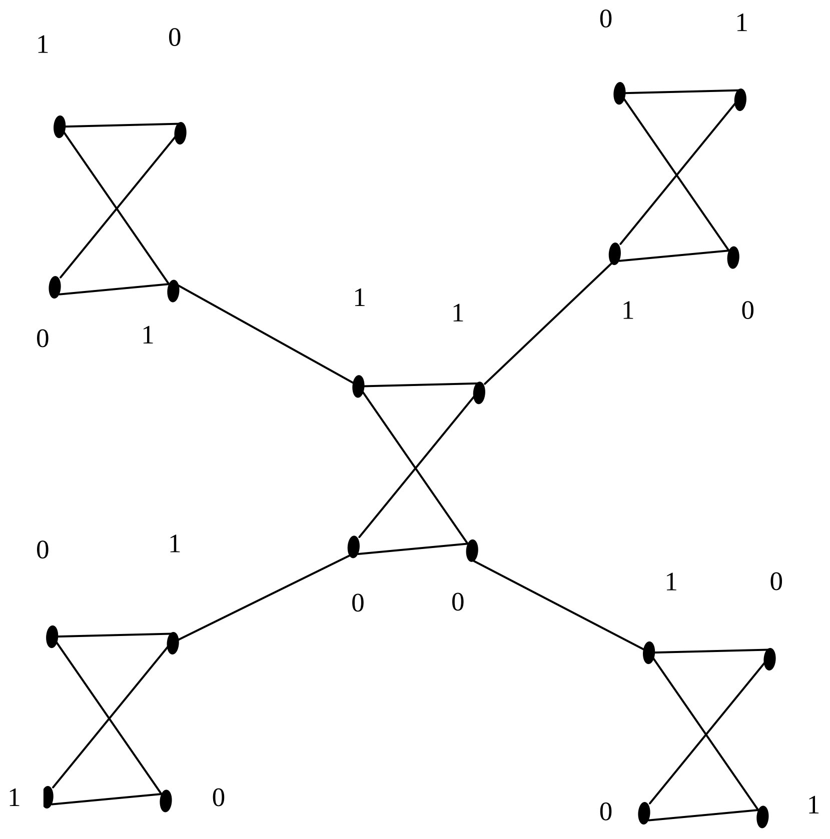

Figure 3.3 : Cordial labeling of star of complete bipartite graph $K_{2,2}$

Here $v_f(0) = 10 = v_f(1)$, $e_f(0) = 12$ and $e_f(1) = 12$.

Thus $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence the above graph is cordial.

.**Illustration 3.4:**

The cordial labeling of star of complete bipartite graph $K_{4,4}$ is shown in figure 3.4 as an illustration for the proof of Theorem 1. It is the case related to $n \equiv 0(mod\ 4)$

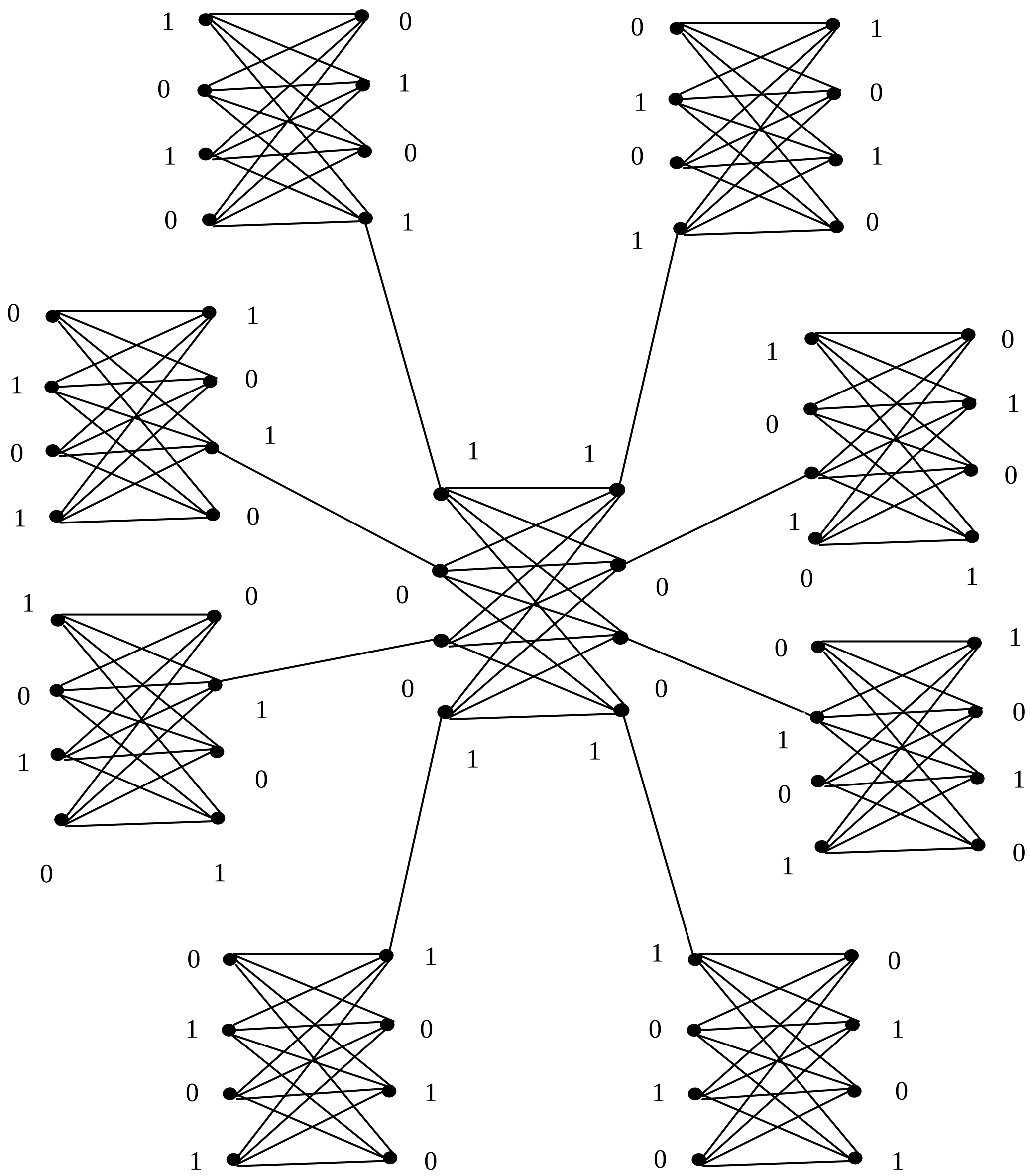

Figure 3.4 : Cordial labeling of star of complete bipartite graph $K_{4,4}$

Here $v_f(0) = 10 = v_f(1)$, $e_f(0) = 12$ and $e_f(1) = 12$.

Thus $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Hence the above graph is cordial.

References:

[1]. Cahit I. , Cordial graphs: A weaker version of Graceful and Harmonic graphs, Ars Combinatorial, 23 (1987) 201-207.

[2]. Cahit I. , On cordial and three-equitable labeling of graphs. Utin.Math. , 37(1990) 189-198.

[3]. Gallian.J.A. , A dynamic survey of graph labeling, The electronic journal of Combinatorics, 19(2012), #DS6 1-260.

[4]. Harary F. , Graph theory, Addition-wesley, Reading, MA, 1969.

[5]. P.Selvaraju[1001], New classes of graphs with α-valuation, Harmonius and Cordial labeling, Ph.D. Thesis, Anna University, 2001. M.K.University, 2002.

[6]. Shee S.C. , Ho Y.S., The cordiality of path-union of n copies of a graph, Discrete Math, 151(1996) 221-229.

[7]. Vaidya S.K., Ghodasara .G.V., Sri Vastav .S., Kaneria V.J., Cordial labeling for two cycle related graphs, The Mathematics Student, 76(2007), 237-246.